MATHEMATICS 20

TEACHER MANUAL

CLASSROOM ASSESSMENT MATERIALS

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TEACHER MANUAL

CLASSROOM
ASSESSMENT
MATERIALS

Alberta
EDUCATION

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The Classroom Assessment Materials

Background

The Classroom Assessment Materials Project (CAMP) was launched in 1994 in response to Alberta Education's goal of establishing and effectively communicating clear learning outcomes and high standards for each area of learning. As well, the project is a response to teachers' ongoing requests for high-quality assessment materials to use in their classrooms. CAMP also addresses the need for a common understanding of provincial standards that is frequently expressed by parents, teachers, school administrators, and other public spokespeople.

Although Alberta teachers and Alberta Education developed the Classroom Assessment Materials for teachers to use in Alberta's schools, educators from other provinces and countries have indicated that these materials have considerable potential for their jurisdictions.

Development

Alberta Education staff have worked closely with teachers from all over Alberta to design and develop the Classroom Assessment materials. A project advisory committee with representation from key education organizations, including the Alberta Teachers' Association, provided essential advice and direction for the overall shape and philosophy of the project. Teachers have contributed in numerous and invaluable ways. They have:

- selected resource material and data bases
- developed questions and activities
- validated materials
- offered their time and classrooms for field testing and pilot testing
- provided advice about administration and manageability
- served on revision committees
- provided advice regarding the articulation of expectations from grade to grade/course to course and across subjects
- written and revised scoring criteria
- selected examples of students' work and written commentaries about them
- confirmed that the standards represented and expressed in the final materials are appropriately demanding, faithful to *Program of Studies* expectations, and clearly expressed or illustrated.

Without the dedication and professionalism of Alberta teachers, this project would not have happened.

Purpose of the Classroom Assessment Materials

The Classroom Assessment Materials are summative assessment packages. They are designed to be used by classroom teachers to assess students' achievement of the learning outcomes specified in the *Program of Studies* relative to clearly stated standards.

The assessment activities in the CAMP materials are designed to be administrated in a classroom setting at times that suit the needs of the teacher and her or his students. The materials are not suitable for any other assessment purpose (e.g., diagnostic assessment, pre-instruction assessment, evaluation of instructional practice, system-wide assessment, program evaluation, teacher evaluation), and therefore they may not be used for any purpose imposed by any authority external to the classroom.

Contents of Each Set of Classroom Assessment Materials

Each set of Classroom Materials contains three "documents":

- a *Teacher Manual* with complete information about the assessment activities, their relation to the *Program of Studies*, the weighting of assessment components, statements of standards, and administration instructions including scoring criteria and details for calculating students' marks
- complete *Student Materials*—all of the information, tests, and booklets that students will need for each component
- *Examples of Students' Responses* that show actual student work in relationship to the scoring criteria, along with explanatory commentary

For each grade, subject, and/or course, there are several assessment components that work together to provide teachers and parents with a broadly based portrait of a student's achievement of the expectations for students learning at the end of that grade/course.

Each set of assessment materials includes a variety of activities—selected-response questions, short written-answer questions, extended writing activities, performance tasks such as lab experiments, problem-solving activities, and oral presentations. All activities are designed to interest students and to be of direct and practical use for teachers. All are directly related to learning outcomes from the *Program of Studies*.

Effective Use of the Classroom Assessment Materials

Teachers may use the Classroom Assessment Materials whenever they want to find out about a student's performance in relation to set standards for the end of that grade/subject/course. The materials were developed with the following questions in mind:

- What knowledge, skills, and attitudes should a student have firmly in place before he or she moves to the next grade or course?
- How well should students completing the learning outcomes for a particular grade/subject/course do what is expected of them?
- What does acceptable work for a grade/subject/course look like?
- What does excellent work for a grade/subject/course look like?

Teachers may administer the components in whatever order suits their classroom assessment needs; however, the components are designed to be used together. Only the complete set of assessment activities will provide a portrait of how well a student has met the standards for that grade/subject/course. Teachers may photocopy the materials as their needs require.

Acknowledgements

This project has come to be because of the remarkable cooperation of school jurisdictions, hundreds of teachers and principals, and thousands of students. From everyone on the project teams—thank you.

The project teams also wish to thank the following organizations without whose consultation and advice the project would not have progressed:

Alberta Teachers' Association
Alberta School Boards' Association
College of Alberta School Superintendents
Alberta Assessment Consortium
Universities Coordinating Council
Association canadienne-française de l'Alberta
Public Colleges and Technical Institutes of Alberta

The Alberta Education CAMP team members from the *Curriculum Standards Brach, Alberta Distance Learning Centre, Language Services Branch,* and the *Student Evaluation Branch*.

CAMP Project Leaders
Frank Horvath and Elana Scraba

CAMP Subject Coordinators

English Language Arts	Elana Scraba
Mathematics	Hugh Sanders
Science	Greg Hall, Greg Thomas, Bernie Galbraith
Social Studies	Doug Burns

Contents

In addition to this *Teacher Manual*, the Mathematics 20 Classroom Assessment Materials includes a complete set of *Student Materials* and *Examples of Students' Responses* in separate booklets.

The Mathematics 20 Classroom Assessment

- *Overview of the Assessment*
- *Components of the Assessment*
- *Blueprint of the Assessment*
- *Description and Terminology*
- *Description of Standards*
- *Relationship of Assessment Standards to Learner Expectations*

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Overview of the Assessment

The NCTM *Assessment Standards for School Mathematics (1995)* states that:

- multiple sources of assessment information should be used to evaluate student progress in mathematics
- there is a shift in the vision of learning mathematics toward investigating, formulating, representing, reasoning, and applying a variety of strategies to the solution of problems
- there is a shift in the role of teachers toward "questioning and listening" and away from "telling" students what to do
- there is a shift in the vision of evaluation toward a system based on evidence from multiple sources

In keeping with this vision, the assessment tasks in this package range from an end-of-course exam including selected-response questions, numerical-response questions, and problem-solving, written-response questions, to tasks that are *projects* and *performance-based*. The projects and performance-based task provide students with an opportunity to demonstrate their understanding of the mathematical process more fully than they are able to on the end-of-course exam.

Type of Assessment	Part/ Activity	Time	Mark Allocation	Percent Allocation
End-of-Course Exam	Selected Response Numerical Response Written Response	2 hr	28 4 8	60
Performance Assessment	Task: Circle Geometry and Trigonometry	0.75 hr	5	15
Project	1. Relations and Functions 2. Math History: A Neglected Domain	min of 4 hrs	30	25
TOTAL		7 hrs	—	100

Mathematics 20

Components of the Assessment

The Mathematics 20 assessment consists of the following components:

- an end-of-course exam consisting of 28 selected-response, 4 numerical-response, and 2 written-response questions
- one performance assessment, which consists of one task, designed to be completed by students individually
- one project designed to be completed by students over a longer period of time. Students may work independently or in groups, collecting data and references, and have input into the details of the project.

Blueprint of the Assessment

	G	P	QE	QF	R	RE	R&F	T
Conceptual Understanding	S/R 23, 24 PA Part 1, 2 Project B	Project B	S/R 18, 20	S/R 16 W/R 2		S/R 5	S/R 11, 12, 14 W/R 1 Project A Project B	S/R 25, 26, 27
Procedural Knowledge	PA Part 1, 2			N/R 3 W/R 2	S/R 1, 2, 3, 4	S/R 6, 7, 8	S/R 13 N/R 1, 2 W/R 1 Project A	N/R 4
Problem Solving	S/R 22 PA Part 1, 2 Project B	S/R 9, 10 Project B	S/R 19, 21 W/R 2 Project B	S/R 15, 17 W/R 2 Project B	Project B	Project B	W/R 1 Project A Project B	S/R 28 Project B

Key

G — Geometry
P — Probability
QE — Quadratic Equations
QF — Quadratic Functions
R — Radicals & Exponents
RE — Rational Expressions
R&F — Relations and Functions
T — Trigonometry

S/R — Selected-Response
W/R — Written-Response
N/R — Numerical-Response
PA — Performance Assessment

Definitions and Terminology

The *Alberta Education Course of Studies* for Mathematics 20 is defined by the learner expectations, what the student is expected to know and can do.

Standard

A *standard* is a reference point used in planning and evaluation. A standard defines the quality of a performance, product, or service that is considered acceptable, or that is considered to be excellent.

In evaluating educational performance, the following forms of standards apply:

- *curriculum* and *assessments standards* apply to the assessment of individual students
- *achievement standards* apply to the assessment of student populations

In this document, only curriculum and assessment standards will be discussed.

Curriculum standard

A *curriculum standard* is a set of learner expectations for a module, course, or grade level of a program. The curriculum standards for Mathematics 20 are defined by the general learner expectations outlined on pages 3 and 4 of the Course of Studies, and the specific learner expectations outlined on pages 5–7 and pages 15–20 of the Course of Studies.

Learner expectations

General learner expectations are concise statements identifying what it is that students are expected to know, to do, and be like upon completion of a module, course, or grade level of a program.

Specific learner expectations are the component knowledge, skills, and attitudes that contribute to general learner expectations. Specific learner expectations identify a range of contexts in which the general learner expectations apply.

Assessment standards

Assessment standards are the criteria used for judging individual student achievement relative to the curriculum standards and are noted in the assessment standard section of this document.

Assessment standards have three components:

- *Criteria* are descriptors of student performance that indicate a standard has been met. Specific criteria may take on the form of a set of descriptors used in assessing performance on a particular task.
- *Examples* are evidence of the quality of student work.
- *Guidelines for assessment and reporting* consist of information and advice for educators, in order to ensure consistency in assessment and reporting.

Assessment Instrument

An *assessment instrument* is a group of questions or tasks given to students to ascertain whether each has met the requirements of the acceptable standard or the standard of excellence.

Blueprint

A *blueprint* is a classification of the questions or tasks making up a particular assessment instrument. The classification may be in terms of standards (acceptable or excellent), content sections (either topics or titles), or mathematical understandings (concepts, procedures, problem solving), or any other such classification.

Scoring Criteria

The *scoring criteria* is a set of descriptions of qualities of work for awarding marks on a particular extended-response question.

Mathematical Understandings

The assessment of students' knowledge of *mathematical procedures* should provide evidence that they can:

- recognize when a procedure is appropriate
- give reasons for the steps in a procedure
- reliably and efficiently execute procedures
- verify the results of procedures empirically (e.g., using models) or analytically
- recognize correct and incorrect procedures
- generate new procedures and extend or modify familiar ones
- appreciate the nature and role of procedures in mathematics

The assessments of students' knowledge and understanding of *mathematical concepts* should provide evidence that they can:

- label, verbalize, and define concepts
- identify and generate examples and non-examples
- use models, diagrams, and symbols to represent concepts
- translate from one mode of representation to another
- recognize the various meanings and interpretations of concepts
- identify properties of a given concept and recognize conditions that determine a particular concept
- compare and contrast concepts

The assessments of students' ability to use mathematics in *solving problems* should provide evidence that they can:

- formulate problems
- apply a variety of strategies to solve problems
- solve problems
- verify and interpret results
- generalize solutions

(From *Curriculum and Evaluation Standards for School Mathematics,* National Council of Teachers of Mathematics, 1989, p. 209, 223, 228.)

Description of Standards

The Course of Studies for Mathematics 20 states that 80% of the content consists of topics required of all students who take the course. The required content contains the concepts, skills, and attitudes that all students are expected to acquire, as well as specific expectations for problem solving and the use of technology. An additional 20% is made up of elective materials, which provide for enrichment, remediation, and innovative or experimental presentations or activities. For complete details of the Mathematics 20 course, refer to the Course of Studies.

The course of studies also states that "evaluation of students in the Senior High School Mathematics program will involve assessment of the level of achievement of **all of the learner expectations, including concepts, skills and attitudes, as well as problem-solving and technological expectations**."

The assessment standards for Mathematics 20 include an acceptable and an excellent level of performance. Students' performance should be measured on a range of tasks, some designed at the acceptable level, some designed at the excellent level, and some designed for assessment at both levels.

Acceptable/Excellent

Students have reached the acceptable standard if they are able to consistently complete acceptable work on routine tasks.

Students have reached the standard of excellence if they are able to consistently complete excellent work on routine tasks and acceptable work on non-routine tasks.

Details of the relationship of assessment standards to curricular learner expectations follow.

Relationship of Assessment Standards to Learner Expectations

Radicals and Exponents

Students can describe, orally and in writing, the relationship between exponential expressions with rational exponents and radical expressions; simplify, add, subtract, and multiply radical expressions.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• change expressions from radical to exponential form and vice versa	
• use the terminology related to radicals to identify the parts of a radical expression	
• use a calculator to approximate the value of radical and exponential expressions	
• use exponential laws to simplify radical and exponential expressions that have numerical bases and radicands	• use exponential laws to simplify radical and exponential expressions that have bases and radicands with variables
• change square root expressions with numerical radicands from mixed to entire radicals and vice versa	• change square root expressions that have radicands with variables from mixed to entire radicals and vice versa
• add, subtract, and multiply square root expressions that have numerical radicands	• add, subtract, and multiply square root expressions that have radicands with variables
• solve and verify radical equations that involve squaring only once to eliminate the radical	• solve and verify radical equations that involve squaring the equation more than once to eliminate the radicals
• participate in and contribute toward the problem-solving process for problems that require the solution to radical equations studied in Mathematics 20	• complete the solution to problems that require the solution to radical equations studied in Mathematics 20

Rational Expressions

Students can describe rational expressions as algebraic fractions; perform the operations of addition, subtraction, multiplication, and division on them; and determine the non-permissible values of the variable in the expressions, graphically and algebraically.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• define, identify, and give examples of rational expressions	
• determine and verify non-permissible values of the variable in rational expressions, graphically and algebraically	
• use technology to verify the existence of non-permissible values in rational functions where the degree of the denominator is greater than the degree of the numerator	
• simplify rational expressions by factoring	
• explain why there are some values of the variable in rational expressions that make the rational expression undefined	
• add, subtract, multiply, and divide two rational expressions	• perform the operation of multiplication and division on up to three rational expressions
	• using order of operations, apply a combination of operations to find an equivalent rational expression
• find solutions, not restricted by any non-permissible values, of equations involving rational expressions	• find solutions of equations involving rational expressions
	• complete the solution to problems that require the solutions to rational expressions

Probability

Students can determine the probability of a simple or compound event occurring, and design and carry out simulations of events that have known and unknown probabilities.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• use the terminology related to probability	
• recognize, give examples, and compare theoretical and experimental probability	
• write the sample space for any experiment that has easily countable sample spaces	
• explain and interpret, orally and in writing, the significance of specific probabilities	
• determine the probability of events that have easily countable sample spaces	• determine the theoretical probability of events not occurring after determining the probability that the events will occur
• recognize, give examples, and compare dependent and independent events	
• determine the experimental and theoretical probability of two independent or dependent events	• determine the experimental theoretical probability of more than two independent events
• recognize, give examples, and compare mutually exclusive and non-mutually exclusive events	
• determine the experimental and theoretical probability of the occurrence of either of two events	
• design and carry out simulations that model events, with known probabilities	• use simulation methods to model events that have unknown probabilities
• explain, orally and in writing, the purpose of simulations that model events with known probabilities	
• recognize that the probability of two mutually exclusive events occurring together is zero	• complete the solution to problems that require the understanding and modelling of probability studied in Mathematics 20

Functions and Relations

Students can model the quantitative relationships of certain observed phenomena, graphically and algebraically.

Acceptable Standard

The student can

- explain and illustrate the concept of relation and a function
- graph and interpret relations describing physical phenomena
- identify the domain and range of relations, graphically and algebraically
- describe the effect on the range of a relation when limitations are placed on the domain
- identify the dependent and independent variables, and determine the domain and range for observed phenomena that can be described as direct, inverse, or partial variations
- use functional notation to describe the quantitative relationship between observed phenomena
- illustrate and recognize linear, identity, constant, quadratic, absolute value, and exponential functions, algebraically and graphically
- draw and analyze the graphs of functions, using calculators or computers
- write and sketch the inverse of a given relation represented as a graph or in an equation
- describe the relationship between the domain and range of a relation and the domain and range of the inverse of the relation
- relate the zeros of a function to the x-intercepts of its graph
- describe and sketch the graphs of $y = cf(x - a) + b$ by applying the transformation effects of a, b, and c on the graph of $y = f(x)$

Standard of Excellence

The student can also

- illustrate and recognize cubic and reciprocal functions, algebraically and graphically
- determine the inverse of a function expressed in function notation
- describe the result of evaluating a function $f(x)$ at its inverse;

  i.e., $f\left(f^{-1}(x)\right) = x$
- write the equation of a transformed graph in the form $y = cf(x - a) + b$, given the graph of $y = f(x)$ and the transformed graph

Quadratic Functions

Students can sketch the graph of, and analyze the characteristics of, a quadratic function.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• recognize and give examples of quadratic functions	
• transform the equation of quadratic functions from the general form $y = ax^2 + bx + c$ to the standard form $y = a(x - h)^2 + k,\ a,\ b,\ c,\quad h,\quad k, I \in$	• transform any quadratic function in general form to standard form and analyze its graph
• approximate the x- and y-intercepts, the coordinates of the vertex, the domain, the range, the maximum or minimum values, and the axis of symmetry of the graph of a quadratic function generated on a calculator or computer	
• determine the x- and y-intercepts, the coordinates of the vertex, the domain, the range, the maximum or minimum values, and the axis of symmetry of an integral quadratic function algebraically	
• relate graphing a quadratic function in the form $y = a(x - h)^2 + k$ to transformations	
• write the equation of the graph of a quadratic function	
• participate in and contribute toward the solution to problems that require the analysis of quadratic functions studied in Mathematics 20	• complete the solution to problems that require the analysis of quadratic functions studied in Mathematics 20

Quadratic Equations

Students can solve quadratic equations and relate their solutions to the x-intercepts of the graph of their corresponding quadratic functions. Students can use quadratic equations to solve problems.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• recognize and give examples of a quadratic equation	
• determine the solutions to any quadratic equation in one variable, and express the solution as an exact value or as an approximate value	
• evaluate the discriminant of a quadratic equation, to determine the roots	
• explain the relationship among the roots of a quadratic equation, the *x*-intercepts of the graph of the corresponding quadratic function, and the zeros of the quadratic function	
• solve and verify equations that contain rational or radical expressions that, when simplified, produce a quadratic equation that can be factored	• solve and verify quadratic equations that contain radicals that involve squaring the equation more than once to eliminate the radicals
	• derive an integral quadratic equation, given its roots, and recognize that the solution is not unique
• participate in and contribute toward the solution to problems that require the use of quadratic equations studied in Mathematics 20	• complete the solution to problems that require the solution to quadratic equations studied in Mathematics 20

Geometry

Students can describe the relationship between the perpendicular bisector of a chord and the centre of the circle; the relationship between the tangent line to a circle and the radius drawn to the point of contact; and solve problems, using these relationships.

Acceptable Standard

The student can

- use the terminology related to a circle
- demonstrate that the perpendicular bisector of a chord passes through the centre of the circle
- describe the relationship between the perpendicular bisector of a chord and the centre of the circle, and write a logical argument to support this relationship
- demonstrate that a perpendicular drawn from the point of contact of a tangent passes through the centre of the circle
- describe the relationship between the tangent line to a circle and the radius drawn to the point of contact, and write a logical argument to support this relationship
- calculate the length of a chord, the distance from a chord to the centre of the circle, and the radius of the circle, given any two of the measures
- calculate, given a point in the exterior of a circle, the length of tangent segments from the point, the lengths of radii to the points of contact of the tangent, and the distance from the point to the centre of the circle, given any two of the measurements
- participate in and contribute toward the problem-solving process required to solve the problems, involving the geometric relationships studied in Mathematics 20

Standard of Excellence

The student can also

- complete the solutions to problems involving geometric relationships studied in Mathematics 20

Trigonometry

Students can determine the sine, cosine, and tangent of any angle in standard position, and solve problems that involve oblique triangles.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• recognize, sketch, and determine the reference angles for positive and negative angles in standard position	
• determine the sine, cosine, and tangent ratios for any angle	
• determine any two values of x, y, r, or θ, of an angle in standard position, given the other two	
• determine the measures of unknown sides and angles in right triangles	
• determine the measures of unknown sides and angles in oblique triangles (find one solution to an oblique triangle involving the ambiguous case)	• determine the measures of unknown sides and angles in oblique triangles, including ambiguous case
• solve problems involving oblique triangles	• show how the right-triangle trigonometric ratios relate to the derivation of the sine law and cosine law

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Administering and Scoring the Assessment Components

- *End-of-Course Exam*
- *Performance Assessment*
- *Projects*

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End-of-Course Exam
Item Summary

ITEM #	UNIT	CURRICULUM EXPECTATION*	TAXONOMY
SR 1	R	1.1	P
2	R	2.2	P
3	R	1.2	P
4	R	2.1, 2.2	P
5	RE	1.1	C
6	RE	1.2	P
7	RE	1.4	P
8	RE	1.5	P
9	P	2.2	PS
10	P	2.2	PS
11	R&F	2.4	C
12	R&F	1.2	C
13	R&F	3.3	P
14	R&F	2.5	C
15	QF	1.3	PS
16	QF	1.3	C
17	QF	1.3.1	PS
18	QE	1.4	C
19	QE	1.3	PS
20	QE	1.3	C
21	QE	1.1.1	PS
22	G	1.2.1	PS
23	G	1.2, 2.2.1	C
24	G	1.2, 2.2	C
25	T	2.1.1, 2.2.2	PS
26	T	1.1	C
27	T	1, 1.4.1	C
28	T	1.4.1	PS
NR 1	R&F	2	P
NR 2	R&F	2.7	P
NR 3	QF	1.2	P
NR 4	T	2.3	P
WR 1	R & F	2.7	PS
WR 2	QF	1.3, 1.1	PS

**Course of Studies*, Mathematics 20

QUESTION TYPE	MARKS	PERCENT EMPHASIS
Selected Response	**28**	**80**
Numeric Response	**4**	
Written Response	**8**	**20**

QUESTION STYLE	MARKS	PERCENT EMPHASIS
Concepts	**11**	**34**
Procedures	**12**	**34**
Problem Solving	**17**	**32**

Key

G—Geometry
P—Probability
QE—Quadratic Equations
QF—Quadratic Functions
R—Radicals & Exponents
RE—Rational Expressions
R&F—Relations and Functions
T—Trigonometry
C—Concept
P—Procedure
PS—Problem Solving

Administration

This requires 2 hours of class time and is a closed-book exam. Students will be given a formula sheet, but must provide their own scientific calculator, ruler, and compass. They are expected to work on their own.

Selected-Response Scoring Key

1.	C	**15.**	B
2.	B	**16.**	C
3.	A	**17.**	A
4.	D	**18.**	A
5.	D	**19.**	A
6.	D	**20.**	B
7.	D	**21.**	A
8.	A	**22.**	D
9.	B	**23.**	A
10.	D	**24.**	B
11.	D	**25.**	B
12.	C	**26.**	C
13.	A	**27.**	D
14.	D	**28.**	D

Numerical-Response Scoring Key

1. 8.0
2. 0.33
3. 3.0
4. 6.4

Note: The Student Booklet contains instructions for answering numerical-response questions on answer sheets with prepared boxes. If this type of answer sheet is not available, these pages should be replaced with appropriate instructions for your students.

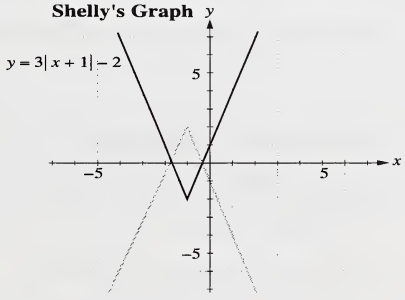
Note: The responses that follow represent **ONE** approach to each of the problems. During your marking, provision should be made for considering the various approaches students may use.

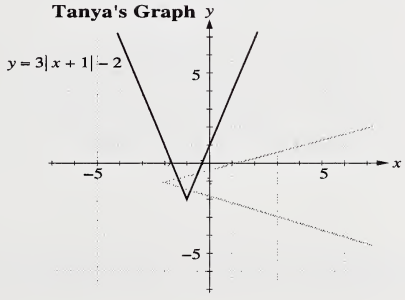
Scoring Written Response 1

Written Response – 4 marks

1. Shelly and Tanya are investigating the absolute value function. Both Shelly and Tanya correctly graph $y = 3|x + 1| \pm 2$, and then they each graph what they believe to be the inverse of this function.

Either Shelly or Tanya constructed a correct graph. Shelly concludes that the inverse of this function is a function, whereas Tanya concludes that the inverse of this function cannot be described by a single function. Identify who is correct, and explain how you reached your conclusion.





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Continued

A Solution for Written Response 1

Tanya's graph is correct because, graphically, the inverse of a relation is the reflection of the relation across the line $y = x$. Tanya reflected $y = 3|x + 1| \pm 2$ across the line $y = x$, whereas Shelly reflected across the x-axis.

Scoring Criteria for Written Response 1

4 marks Tanya's graph is determined to be correct, and a correct reason is provided.

3 marks Tanya's graph is determined to be correct. An insufficient reason is provided. For example, the student describes only one point on the relation and its inverse. ($(-1, -2)$ and $(-2, -1)$).

2 marks Tanya's graph is determined to be correct, and an attempt is made to provide a reason.

1 mark The student demonstrates some understanding of an inverse. For example, students make a statement like "interchange x and y."

OR

Shelly's graph is determined to be correct. The reasoning includes true statements about Shelly's graph.

0 marks The student is off topic, or leaves the paper blank.

Note: See also *Examples of Students' Responses*, included in this assessment package.

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Scoring Written Response 2

Written Response – 4 marks

2. Most linear measurements in Canada are represented by a metric unit such as the metre. However, Canadian football continues to measure in yards. While watching a football game, Justin views an opening kick. The path that the football travels is parabolic, reaching a maximum height of 12 yards. When the football lands, it has travelled 60 yards down the field.

- Sketch the path that the football travels on a coordinate system. Assume that the origin represents the place where the football is kicked. Points plotted on the coordinate system should be of the form
  (distance football travels downfield, height of football).

- Identify the position of the horizontal intercepts and the vertex, and label the graph.

Scoring Criteria for Written Response 2

2 marks	The student draws the sketch of a parabola that correctly models all the given information and, in some way, identifies the vertex, the horizontal intercepts, and labels the axes.
1 mark	The student draws the sketch of a parabola that correctly models all the given information but fails to label the sketch correctly. .
0 marks	The student is off topic, or leaves the paper blank.

- Determine the equation of the relation between the height of the ball and its distance down the field from the kickoff point.

A Solution for Written Response 2

$y = -\frac{1}{75}(x-30)^2 + 12$ where x is the distance the football travels in yards and y is the height reached by the football.

Scoring Criteria for Written Response 2

2 marks	The student obtains the equation $y = -\frac{1}{75}(x-30)^2 + 12$ or an equivalent form, or an equation that corresponds to the student's graph.
1 mark	The student demonstrates some understanding of the relationship between the graph and the equation. .
0 marks	The student is off topic, or leaves the paper blank.

Note: See also *Examples of Students' Response* included in this assessment package.

Performance Assessment

Administration

Overview

Using concepts from circle geometry and trigonometry, students determine the range of vision a person has on a curved planet.

Learner expectations

The student is expected to

- determine the relationship between the tangent line to a circle and the radius drawn to the point of contact
- determine any two values of x, y, r, or θ of an angle in standard position, given the other two
- participate in and contribute toward the problem-solving process required to solve problems involving the geometric relationships studied in Mathematics 20

Student prerequisite knowledge and skills

Completion of circle geometry and trigonometry

Performance

Individual

Time required

45 minutes maximum

Materials required

- one lid from a 4 L ice cream pail per student
- a piece of heavy thread or string about 40 cm long
- one needle per student

Scoring Criteria

A copy of the *Scoring Criteria* for the performance assessment is included in the *Student Booklet*. This should be given to students prior to their completion of this part of the assessment package.

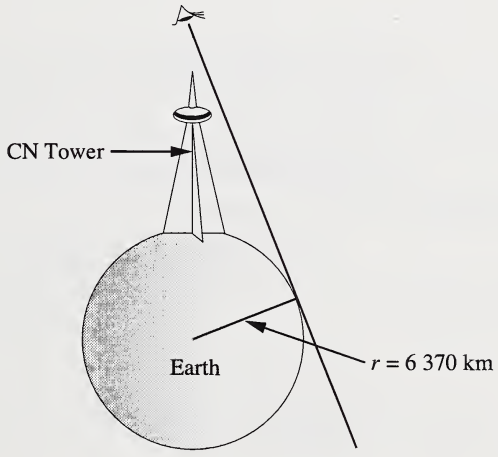
Student Task

According to the song, *"on a clear day you can see forever"* ; however, on a curved planet this is **not** true!

The following tasks are concerned with the range of vision of an observer on a curved planet.

In order to assist you in modelling parts of the following tasks, obtain from your teacher:
- 1 ice cream pail lid
- a needle
- a piece of thread or string

Suppose that from the top of the CN Tower, which is 400 m high, your line of vision is as shown below. Earth has a radius of 6 370 km.



- Calculate the farthest distance that can be seen from the top of the CN Tower. A model simulating this situation may assist you with your solution.

- Suppose the farthest distance a person standing on level ground can see is 4.6 km. How many metres above the ground is the person's line of vision?

- Derive a formula expressing the maximum distance seen (d), in terms of the radius of a planet (R), and the height of the observer (h).

Possible Solution

- Let C be the centre of Earth, T the person at the top of the tower, and H the farthest point seen on the horizon.

$$\begin{aligned} \left(\overline{TC}\right)^2 &= \overline{HT}^2 + \overline{CH}^2 \\ (6\,370.4)^2 - (6\,370)^2 &= \left(\overline{HT}\right)^2 \\ 71.4 \text{ km} &\approx \overline{HT} \end{aligned}$$

The farthest distance seen from the top of the CN tower is approximately 71 km.

- $$\begin{aligned} \left(\overline{TC}\right)^2 &= \overline{HT}^2 + \overline{CH}^2 \\ \overline{TC}^2 &= 4.6^2 + 6\,370^2 \\ \overline{TC} &= 6\,370.001661 \\ 6\,370.001661 - 6\,370 &= 0.001661 \end{aligned}$$

  approximately 1.7 m high.

- $$\begin{aligned} (R+h)^2 &= R^2 + d^2 \\ R^2 + 2hR + h^2 &= R^2 + d^2 \\ \sqrt{2hR + h^2} &= d \end{aligned}$$

Note: See also *Examples of Students' Responses*, included in this assessment package.

Mathematics 20

Scoring Criteria for Performance Assessment	
5 marks	*The student* • demonstrates an understanding of the problem. • uses appropriate mathematical knowledge and problem-solving techniques to find the solution • justifies the solution
4 marks	*The student* • demonstrates an understanding of the problem • uses appropriate mathematical knowledge and problem-solving techniques to find a solution, but the solution contains a minor flaw • shows some justification for results
3 marks	*The student* • demonstrates some understanding of the problem • uses appropriate mathematical knowledge and problem-solving techniques to find *partial* solutions • communicates little understanding of the complexities of the problem • formulates some aspects of the problem mathematically
2 marks	*The student* • explores the initial stages of the problem • applies some relevant mathematical knowledge and problem-solving techniques to find *partial* solutions
1 mark	*The student* • applies some relevant mathematical knowledge to the problem
0 marks	*The student* • is off topic, or leaves the paper blank.

Projects

Background

What is a project?

Projects provide students with opportunities, not normally available in an end-of-course examination, to carry out independent investigations of situations, problems, or issues in a mathematical context.

Characteristics of a project

- Projects are completed over a longer period of time.
- Students work independently or in groups, collecting their own data and references.
- Students are encouraged to use mathematical tools such as computerized spreadsheets, to which they may not have been formally exposed in class.
- Students decide some of the details of their projects.

Although the project format is intended to encourage students to work independently, the degree to which this occurs will depend upon the maturity of the student and the purpose of the project. Teachers are involved in advising students on project plans, in checking the availability and appropriateness of resources, in monitoring student progress, and in assessment.

Why include projects in a mathematics course?

Projects enhance the quality of learning in mathematics. Since students are involved in choosing significant aspects of their projects, they are likely to be more motivated and the quality of their learning is enhanced. Independent work by students helps them develop important skills of self-organization and responsibility, as well as practising communication of mathematical ideas through report writing.

Administration of the Project

The project can be completed either in or outside of class time.

Students may choose from **either** Project A **or** B.

Before students start the project, you should discuss:

- The nature of the project
- The conditions under which the task must be completed; i.e., timelines, group work, etc.
- The *Scoring Criteria*, which are included in the student package.

- The format for the report.
  - The more input students have into this aspect, the higher will be their level of motivation. There is a sample outline provided so that you can begin your discussion with your students.

- How the project will be assessed.
  - Sample scoring criteria are provided, but modification with student input is encouraged.

In order to help determine that the work is a student's own, you may wish to require that students provide evidence of their ongoing work on the project, by setting dates for submission of their project plans, rough drafts, etc.

If students work in groups, either because the project requires more work than can reasonably be done by an individual student or because some specific interaction between students is required, the reason that the project is being conducted by a group and the individual student contribution should be clearly documented in the report. Each student, individually, should be required to complete the analysis, interpretation, and writing-up stages.

The project report should be written as concisely as possible. Instruct students to limit the main text to 10 pages.

Mathematics 20

Scoring Criteria for Project					
Scoring Categories	**Not Shown** ( ) marks	**Low** ( ) marks	**Medium** ( ) marks	**High** ( ) marks	**Comments**
Defining the Project (10%)	(0)	(1)	(2)	(3)	
Conducting the Project (20%)	(0)	(2)	(4)	(6)	
Analysis (20%)	(0)	(2)	(4)	(6)	
Mathematical Content (30%)	(0)	(3)	(6)	(9)	
Communication (20%)	(0)	(2)	(4)	(6)	

Total marks: (30) _____%

Using the Scoring Criteria

A student's project will be rated on several categories; and the scores summed from each category to obtain a total mark for the entire project. This type of scoring, referred to as *anaholistic*, provides more information about a task and furnishes information from different perspectives, pinpointing strengths and weaknesses. The *Scoring Criteria* has three categories that relate specifically to parts of the project. These are *Defining the Project, Conducting the Project,* and *Analysis*. The last two categories, *Mathematical Content* and *Communication*, are intended to be used as overall assessment categories for the entire project.

In the body of the table, possible marks for each category are indicated in brackets. For example, the total possible marks for the category *Defining the Project* is 3, whereas the total possible marks for *Mathematical Content* is 9. Each category has a continuum of marks. For example, a student may receive a mark anywhere from 0 to 9 on the *Mathematical Content*. The marks allotted in each of the five sections of categories are summed for a possible total of 30.

The detailed categories for the *Scoring Criteria* are on the next page.

Scoring Categories for Mathematics 20: Projects A and B	*Ratings*
Defining the Project: • identifying information required • asking useful questions • acknowledging difficulties and limitations • suggesting extensions • statement of project topic • statement of purpose of project	3 – thoroughly evident 2 – substantially evident 1 – minimally evident 0 – not shown
Conducting the Project: • collecting appropriate factual information • consulting appropriate resources (peers, teachers, references) • organizational skills	6 – exemplary 4 – effective 2 – marginal 0 – not evident
Analysis: • analyzing information • interpreting and critically evaluating the mathematicians' contribution to mathematics • drawing logical conclusions • evaluation of conclusions	6 – thoroughly developed 4 – partially developed 2 – minimally developed 0 – not developed
Overall Categories	***Ratings***
Mathematical Content: • connection of the mathematics researched to our curriculum • use of mathematical language and symbols • mathematical formulation or interpretation of problem • interpretation and evaluation of mathematics used	9 – proficient 6 – adequate 3 – marginal 0 – unsatisfactory
Communication: • coherent presentation • information clearly presented • references cited • acknowledgment of assistance • appendices	6 – clear, well-organized, complete 4 – mostly clear and well-organized 2 – some clear parts 0 – unclear and disorganized

Mathematics 20

Calculating and Recording Student Achievement

- *Standards for Overall Performance on the Assessment*
- *Class Record Form*
- *Directing Words*

CLASSROOM
ASSESSMENT
MATERIALS

Standards for Overall Performance on the Assessment

Scoring criteria for each component of the assessment are provided in the teacher instructions for each component. Assessment standards for the overall performance of a student on the whole package are as follows:

Not Yet At Acceptable Standard (NS)	Acceptable Standard (AC)	Standard of Excellence (EX)
0%–49% on the total score	**50%–79% on the total score**	**80%–100% on the total score**

Standards for the assessment were established and validated using the scoring criteria outlined for each task. If the tasks are scored using different criteria or used for purposes other than assessing achievement at the end of Mathematics 20, these standards may not be appropriate.

The standards for performance on the assessment apply to all students. Professional judgement should be used to make adjustments to administration procedures so that special needs students are able to demonstrate their best work.

Mathematics 20

Class Record Form

Student Name	Exam		Performance Assessment		Project		Overall	Standard*
	End-of-Course Examination (%)	(A) % multiplied by 0.6	%	(B) % multiplied by 0.15	%	(C) % multiplied by 0.25	Add columns (A) and (B) and (C)	

*The standards for this assessment package are as follows:

Not Yet at Acceptable Standard (NS)	Acceptable Standard (AC)	Standard of Excellence (EX)
0%–49%	50%–79%	80%–100%

Directing Words

Discuss
The word "discuss" will not be used as a directing word on math and science diploma examinations because it is not used consistently to mean a single activity.

The following words are specific in meaning:

Contrast/Distinguish
Point out the *differences* between two things that have similar or comparable natures.

Compare
Show the character or relative values of two things by pointing out their similarities and differences.

Conclude
State a logical end based on reasoning and/or evidence.

Criticize
Point out the merits and demerits of an item or issue.

Define
Provide the essential qualities or meaning of a word or concept. Make distinct and clear by marking out the limits.

Describe
Give a written account or represent the characteristics of something by a figure, model, or picture.

Design/Plan
Construct a plan, i.e, a detailed sequence of actions, for a specific purpose.

Enumerate
Specify one by one or list in concise form and according to some order.

Evaluate
Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages.

Explain
Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail.

How
Show in what manner or way, and with what meaning.

Hypothesize
Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically.

Identify
Recognize and select as having the characteristics of something.

Illustrate
Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram.

Infer
Form a generalization from sample data; arrive at a conclusion by reasoning from evidence.

Interpret
Tell the meaning of something; present information in a new form that adds meaning to the original data.

Justify/Show How
Show reasons for or give facts that support a position.

Outline
Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps.

Predict
Tell in advance on the basis of empirical evidence and/or logic.

Prove
Establish the truth, validity, or genuineness of something by giving factual evidence or logical reasons.

Relate
Show logical or causal connection between things.

Solve
Give a solution for a problem; i.e., explanation in words and/or numbers.

Summarize
Give a brief account of the main points.

Trace
Give a step-by-step description of the development.

Why
Show the cause, reason, or purpose.

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